Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

CALCULUS.

---

75. Proposed by O. W. ANTHONY, M. Sc., Instructor in Mathematics, Boys' High School, New York City.

Solve the differential equation $\frac{d^2y}{dx^2}+n^2y=\frac{6}{x^2}y$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $y=ux^3$, the equation then becomes $\frac{d^2u}{dx^2}+\frac{6}{x}.\frac{du}{dx}+n^2u=0$.

Consider the equation $d^2v/dx^2+n^2v=0$.

Then $(dv/dx)^2+n^2v^2=A=n^2c^2$.

$\therefore\ \pm dv/\sqrt{(C^2-v^2)}=ndx$. $\therefore\ v=c\cos(nx+\alpha)$, or $c\sin(nx+\alpha)$.

Let $x^2=dz$, and change the independent variable from x to z.

$\therefore\ 2z(d^2v/dz^2)+dv/dz+n^2v=0$.

Differentiate this last three times with regard to z, and let $t=d^3v/dx^3$, then we get $2z.(d^2t/dz^2)+7.(dt/dz)+n^2t=0$.

Now re-change the independent variable from z to x.

$$\therefore\ d^2t/dx^2+\frac{6}{x}.\frac{dt}{dx}+n^2t=0.$$

$$\therefore\ u=t=\frac{d^3v}{dz^3}=\left(\frac{1}{x}.\frac{d}{dx}\right)^3[c\cos(nx+\alpha)].$$

$$\therefore\ y=x^3\left(\frac{1}{x}.\frac{d}{dx}\right)[c\cos(nx+\alpha)],$$

$$=cn^3\left[\left(1-\frac{3}{n^2x^3}\right)\sin(nx+\alpha)+\frac{3}{nx}\cos(nx+\alpha)\right],$$

$$=C\left[\left(1-\frac{3}{n^2x^2}\right)\sin(nx+\alpha)+\frac{3}{nx}\cos(nx+\alpha)\right].$$

If we had written $v=c\sin(nx+\alpha)$,

$$y=C\left[\frac{3}{nx}\sin(nx+\alpha)-\left(1-\frac{3}{n^2x^2}\right)\cos(nx+\alpha)\right].$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

This is a "case" of Riccati's Equation of the form

$$\frac{d^2u}{dx^2}-a^2x^{2q-2}u=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

and of scientific interest as it appears in the theory of the figure of the earth. See Airy's *Figure of the Earth*, articles 64, 65; Col. Clark's *Geodesy*, page 80; Tisserrand's *Méc. Cel.*, *Tome* 2, Chapitre xiv *Equilibre d'Une Masse Fluide Hétérogène*, in the last of which the equation is

$$\frac{d^2y}{dx^2}=\left\{\frac{a^2\dfrac{d\rho}{da}}{\displaystyle\int_0^a \rho a^2 da}+\frac{6}{a^2}\right\}y \text{ ....(2).}$$

It is due to this connection, that the proposed equation has received a variety of solutions.

What is here given is based on a transformation of (1) and a pair of solutions given in Johnson's *Differential Equations*, articles 213, 214.

(1) is transformed into

$$\frac{d^2u}{dx^2}-\frac{m-1}{x}\frac{du}{dx}-a^2u=0 \text{ .... (3),}$$

two integrals of which are of the form

$$u_3=e^{ax}\left(1-\frac{m-1}{m-1}ax+\frac{(m-1)(m-3)}{(m-1)(m-2)}\frac{a^2x^2}{2\,!}-\dots\right) \text{ ....(4),}$$

$$u_5=e^{-ax}\left(1+\frac{m-1}{m-1}ax+\frac{(m-1)(m-3)}{(m-1)(m-2)}\frac{a^2x^2}{2\,!}+\dots\right) \text{ ....(5),}$$

and connected with u by the relation $u=Au_3+Bu_5$ ....(6).

Now assuming $u=x^pv$.........(7), or $v=ux^{-p}$.........(8), (3) becomes

$$\frac{d^2v}{dx^2}-a^2v=\frac{p(p+1)v}{x^2} \text{ ....(9).}$$

Putting $a=n\surd(-1)$, $p=2$, $m=2p+1=5$, and $v=y$, $\surd(-1)=i$, (6) and (8), with (4) and (5) give

$$y=ux^{-2}=x^{-2}[Ae^{nix}+Be^{-nix}-\tfrac{1}{3}n^2x^2(Ae^{nix}+Be^{-nix})+nx(Bie^{-nix}-Aie^{nix})] \text{ ...(10),}$$

which it is not difficult to reduce to the form,

$$y=Cx^{-2}[(3-n^2x^2)\cos(nx+\alpha)+3nx\sin(nx+\alpha)] \text{ ....(11).}$$

See Forsyth's *Differential Equations*, Ed. 1885, Ex. 3, page 65; Ex. 1, page 175; Ex. 21, page 180; Ex. 26, page 181, and Ex. 5, pages 233, 4.

76. Proposed by E. B. ESCOTT, Cambridge, Mass.

Solve the partial differential equation, $q^2r+4pqs+p^2t+p^2q^2(rt-s^2)=a^2$. [*Forsyth's Differential Equations*, page 376.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The equation in λ is, $\lambda^2p^2q^2(1+a^2)+4\lambda p^3q^3+p^4q^4=0$.

$\therefore\ \lambda^2(1+a^2)+4\lambda pq+p^2q^2=0$.

$\therefore\ \lambda=-\{pq/(1+a^2)\}[1\mp\surd(3-a^2)]=m_1pq$ or m_2pq.

The first system of integrals is $p^2q^2dy+m_1p^3qdr+m_1p^3q^3dp=0$.

$p^2q^2dx+m_2pq^3dy+m_2p^3q^3dq=0$, or $qdy+m_1pdx+m_1pq^2dp=0$..........(1);